

Approximate Angle Trisection

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It is well-known that it is impossible to trisect an arbitrary angle using only ruler and compass (this has been proven by P. L. Wantzel (1836)). However, elegant approximate methods, which offer sufficient accuracy for a reasonable constructing effort, have attracted much attention in the scientific community. Our method for approximate angle trisection is presented in Fig.1.

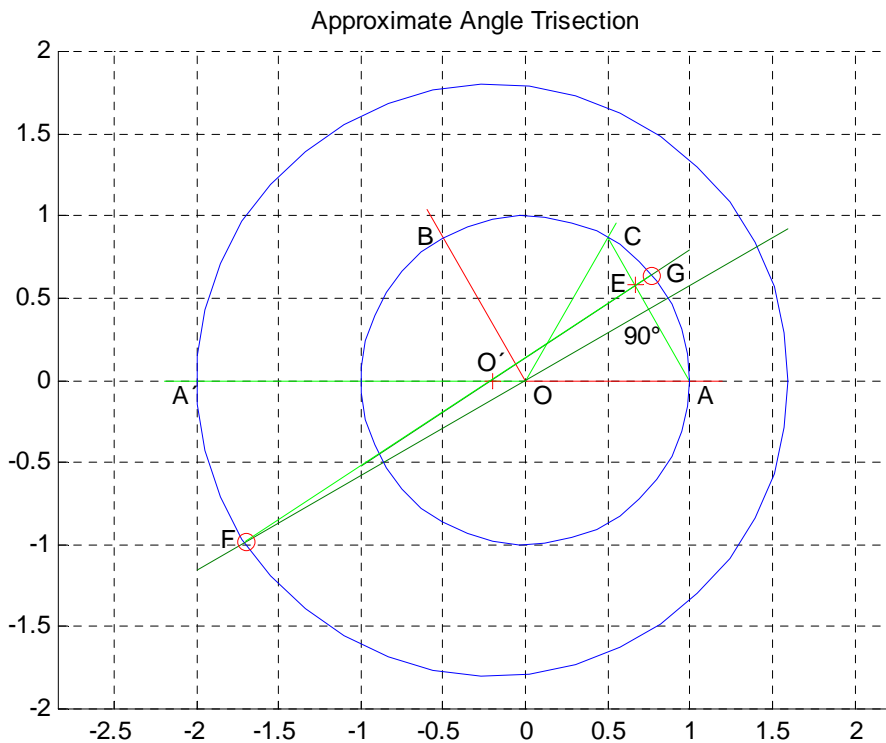


Fig. 1

Let $\angle AOB = \alpha$ be the given angle to be trisected. Draw the bisector $\angle AOC = \alpha/2$, with $\overline{OC} = \overline{OA} = \overline{OB} = 1$. Construct point E on the segment \overline{CA} such that $\overline{CE} = 1/3\overline{CA}$. Construct point A' on the extension of line AO such that $\overline{OA'} = 2\overline{OA}$. Construct point O' on the extension of line AO such that $\overline{OO'} = 1/5\overline{OA}$. Draw a circle $Circ(O', \overline{O'A'})$ with center at O' and radius $\overline{O'A'}$. Draw a line perpendicular to the line CA through point O (or equivalently a bisector of $\angle AOC$) which intersects $Circ(O', \overline{O'A'})$ at point F. Draw the line FE which intersects $Circ(O, \overline{OA})$ at point G. Now $\angle AOG = \varphi$ is an approximate trisection of $\angle AOB$.

In Fig.2 is shown the relative error for $\alpha \in (0^\circ, 180^\circ]$.

Table1 shows some numeric results of the presented trisection.

Fig.2

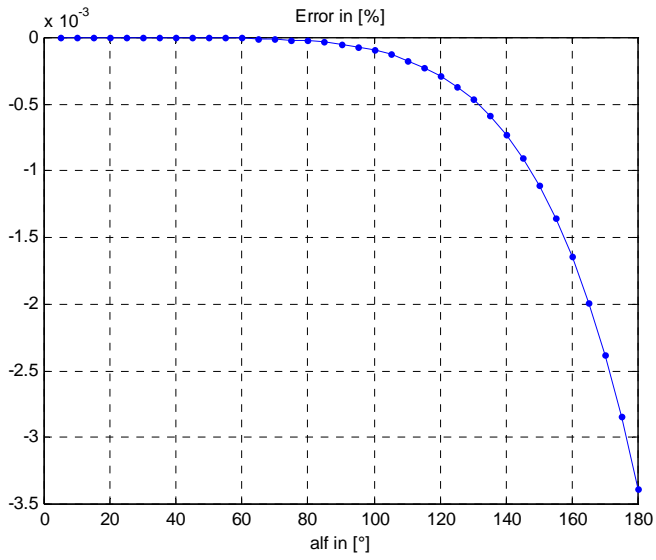


Table1

α in [°]	φ in [°]
5	1.666666666666664
10	3.333333333333022
15	4.99999999994673
20	6.66666666626746
25	8.33333333142897
30	9.9999999317277
35	11.66666664656937
40	13.33333328211942
45	14.9999988310304
50	16.66666642204531
55	18.33333285616349
60	19.9999912165747
65	21.66666512670158
70	23.33333074299084
75	24.99999579569061
80	26.66666005163218
85	28.33332320559168
90	29.99998486448245
95	31.66664452928435
100	33.33330157452529
105	34.99995522512207
110	36.66660453037818
115	38.33324833492694
120	39.99988524639680
125	41.66651359956497
130	43.33313141675330
135	44.99973636420633
140	46.66632570417838
145	48.33289624244081
150	49.99944427090451
155	51.66596550503604
160	53.33245501572618
165	54.99890715525168
170	56.66531547694856
175	58.33167264819404
180	59.99797035626992