A Derivation of Mollweide Equations

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Mollweide equations will be derived using homogenous trilinear coordinates.

In Fig. 1 is shown an arbitrary triangle ABC. Point O is the circumcenter, and C_1, C_2 are the first and second intersection of the circumcircle with the bisector of the side AB.

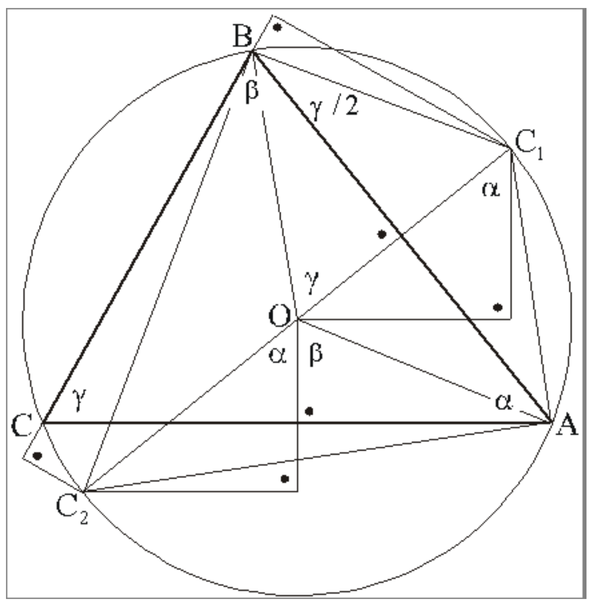

Fig. 1

Mollweide equations are as follows

$$(a+b):c = \cos\frac{\alpha-\beta}{2} : \sin\frac{\gamma}{2}\ , \tag{1}$$

$$(a-b):c = \sin\frac{\alpha-\beta}{2} : \cos\frac{\gamma}{2}\ . \tag{2}$$

From Fig. 1 we obtain the following trilinear coordinates of point C_1

$$C_1 = \cos\beta+\cos\alpha\ :\cos\alpha+\cos\beta\ :\cos\gamma-1\ , \tag{3}$$

and equivalently

$$C_1 = \sin(\beta+\frac{\gamma}{2}) : \sin(\alpha+\frac{\gamma}{2}) : -\sin\frac{\gamma}{2}\ . \tag{4}$$

Since $\beta+\gamma/2=(\beta-\alpha)/2+\pi/2$ and $\alpha+\gamma/2=(\alpha-\beta)/2+\pi/2$, from (4) follows

$$C_1=\cos\frac{\beta-\alpha}{2}:\cos\frac{\alpha-\beta}{2}:-\sin\frac{\gamma}{2}\ . \tag{5}$$

We recall the following trivial relations

$$a=b\ \cos\gamma+c\ \cos\beta\ , \tag{6}$$

$$b=c\ \cos\alpha+a\ \cos\gamma\ . \tag{7}$$

Adding (6) and (7) and after some rearrangement we obtain

$$(a+b)(1-\cos\gamma)=c(\cos\alpha+\cos\beta)\ , \tag{8}$$

or

$$(a+b):c=\cos\alpha+\cos\beta\ :1-\cos\gamma\ . \tag{9}$$

Now comparing (3), (5) and (9) we obtain equation (1).

From Fig. 1 we obtain the following trilinear coordinates of point C_2

$$C_2=\cos\alpha-\cos\beta\ :\cos\beta-\cos\alpha\ :1+\cos\gamma\ , \tag{10}$$

and equivalently

$$C_2=-\cos(\beta+\frac{\gamma}{2}):-\cos(\alpha+\frac{\gamma}{2}):\cos\frac{\gamma}{2}\ . \tag{11}$$

Since $\beta+\gamma/2=(\beta-\alpha)/2+\pi/2$ and $\alpha+\gamma/2=(\alpha-\beta)/2+\pi/2$, from (11) follows

$$C_2=\sin\frac{\beta-\alpha}{2}:\sin\frac{\alpha-\beta}{2}:\cos\frac{\gamma}{2}\ . \tag{12}$$

Subtracting (7) from (6) and after some rearrangement we obtain

$$(a-b)(1+\cos\gamma)=c(\cos\beta-\cos\alpha)\ , \tag{13}$$

or

$$a-b:c=\cos\beta-\cos\alpha\ :1+\cos\gamma\ . \tag{14}$$

Now comparing (10), (12) and (14) we obtain equation (2).